Journal of Combinatorics, Graph Theory and Applications
Vol. 1 No. 1 (January-June, 2026)

Received: 07th July 2025
Revised: 25th October 2025
Accepted: 16th November 2025

Corresponding editor: LiMin Yang

SEMI NEIGHBOURLY IRREGULAR GRAPHS

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Abstract. A connected graph G is said to be semi neighbourly irregular (SNI) if no two adjacent vertices of G have the same number of vertices at a distance two away from them. That is, $d_2(u) \neq d_2(v)$, for every edge uv in $E(G)$, (where $d_2(v)$ is defined as the number of vertices at a distance 2 away from v). This paper suggests a method to construct semi neighbourly irregular graphs which contain every graph of order $n \geq 2$ as an induced subgraph. We have also defined strict semi neighbourly irregular tree and semi neighbourly regular strength of a graph. We also study few properties possessed by semi neighbourly irregular graphs and semi neighbourly irregular trees.

1. Introduction

In this paper, we consider only finite, simple, connected graphs. We follow graph theoretic terminology proposed in the works of J. A. Bondy and U.S.R. Murty [3] and Harary [4]. We denote the vertex set and edge set of a graph G by $V(G)$ and $E(G)$ respectively. The degree of a vertex v is the number of edges incident at v. A graph G is regular if all its vertices have the same degree.

For a connected graph G, the distance $d(u, v)$ between two vertices u and v is the length of a shortest (u, v) path. Therefore, the degree of a vertex v is the number of vertices at a distance 1 from v, and it is denoted by $d(v)$. The set of all verticees at a distance one from v is called the neighbourhood of v and is denoted by $N(v)$. This observation suggests a generalization of degree. That is, $d_d(v)$ is defined as the number of vertices at a distance d from v. Hence $d_1(v) = d(v)$ and $N_d(v)$ denote the set of all vertices that are at a distance d away from v in a graph G. Hence $N_1(v) = N(v)$. Girth of a graph is the smallest cycle in the graph and diameter of graph $G = max\{d(u, v)/u, v \in V(G)\}$. For any $S \subseteq V(G)$, $G(S)$ is the subgraph of G induced by the vertices of S.

A graph G is said to be distance d-regular [2] if every vertex of G has the same number of vertices at a distance d from it. If every vertex of G has exactly k number of vertices at a distance d from it, then we call this graph by (d, k)-regular graph. That is, a graph G is said to be (d, k)-regular graph if $d_d(v) = k$, for all v in G. The $(1, k)$-regular graphs are nothing but k-regular graphs. A graph G is $(2, k)$-regular if $d_2(v) = k$, for all v in G. We observe that $(2, k)$-regular and k-semi regular graphs are same thing. The concept of the semi-regular graph was introduced and studied by Alison Northup [1]. A graph G is said to be k-semi

2000 *Mathematics Subject Classification.* Primary 05C12.

Key words and phrases. Neighbourly irregular, semiregular graph, distance degree regular graph, $(2, k)$-regular graph, induced subgraph.

regular graph if each vertex of G is at distance two away from exactly k vertices of G. That is, if $d_2(v) = k$, for all v in G. Helm graph is obtained from a Wheel by attaching a pendant edge at each vertex of the n-cycle. Webgraph is obtained by joining the pendant points of a helm to form a cycle and then adding a single pendant edge to each vertex of this outer cycle.

A connected graph G is neighbourly irregular graph (NI) if no two adjacent vertices of G have the same degree. This concept was studied by S. Gnana Prakasam and S.K. Ayyaswamy [5]. Above definitions motivate us to define the concept of semi neighbourly irregular graphs (abbreviated as SNI graph), semi neighbourly regular strength of graphs and strict semi neighbourly irregular tree.

An induced subgraph of G is a subgraph H of G such that $E(H)$ consists of all edges of G whose end points belong to $V(H)$. In 1936, Konig [7] proved that if G is any graph whose largest degree is r, then it is possible to add new points and to draw new lines joining either two new points or a new point and an old point, so that the resulting graph H is a regular graph containing G as an induced subgraph. In 1963, Paul Erdos and Paul Kelly [6] determined the smallest number of new vertices which must be added to a given graph G to obtain such a graph. These results motivate us to construct a semi neighbourly irregular graph containing given graph as an induced subgraph.

2. Semi Neighbourly Irregular Graphs

Definition 2.1. A connected graph G is said to be semi neighbourly irregular (SNI) if no two adjacent vertices of G have the same number of vertices at a distance two away from them. That is, $d_2(u) \neq d_2(v)$, for all uv in $E(G)$, where $d_2(v)$ denote the number of vertices at a distance two away from v in G.

Example 2.2. i. Any complete bipartite graph $K_{m,n}$ is semi neighbourly irregular graph only when $m \neq n$.

ii. P_3 is semi neighbourly irregular.

iii. Any complete tripartite graph $K_{m,n,p}$ is semi neighbourly irregular graph only when $m \neq n \neq p$.

Theorem 2.3. *Any complete $m-$ partite graph $K_{n_1,n_2,\ldots,n_m}$ is semi neighbourly irregular if and only if $n_1 \neq n_2 \neq n_3 \neq \ldots n_m$.*

Proof. Let $G(V, E)$ be a complete $m-$ partite graph with partition $(V_{n_1}, V_{n_2}, \ldots, V_{n_m})$ of V. That is, for $i = 1, 2, \ldots, m$, every vertex in vertex set V_i is adjacent to all other vertices in the remaining $m - 1$ sets and non-adjacent with vertices in the same set V_i. Hence every vertex in V_{n_i} is at the distance two away from $n_i - 1$ vertices, $i = 1, 2, 3, \ldots, m$. For any vertex $v \in V_{n_i}$, $d_2(v) = n_{i-1}$, $i = 1, 2, \ldots, m$. Let u, v be two adjacent vertices of G. Then u and v are in two different partition sets of V. Let $u \in V_{n_i}$ and $v \in V_{n_j}$, where $i \neq j$. G is semi neighbourly irregular if and only if $d_2(u) \neq d_2(v)$, $uv \in E(G)$ if and only if $n_i - 1 \neq n_j - 1$, $i \neq j$ if and only if $n_i \neq n_j$, $i \neq j$. Hence $K_{n_1,n_2,\ldots,n_m}$ is semi neighbourly irregular if and only if $n_1 \neq n_2 \neq n_3 \neq n_m$. □

Remark 2.4. The graph $K_{n_1,n_2,\ldots,n_m}$ such that all n_i's are distinct is neighbourly irregular and semi neighbourly irregular. Figure 1(a) illustrates "neighbourly irregular graph need not be semi neighbourly irregular" and Figure 1(b),(c) illustrates "semi neighbourly irregular graph need not be neighbourly irregular".

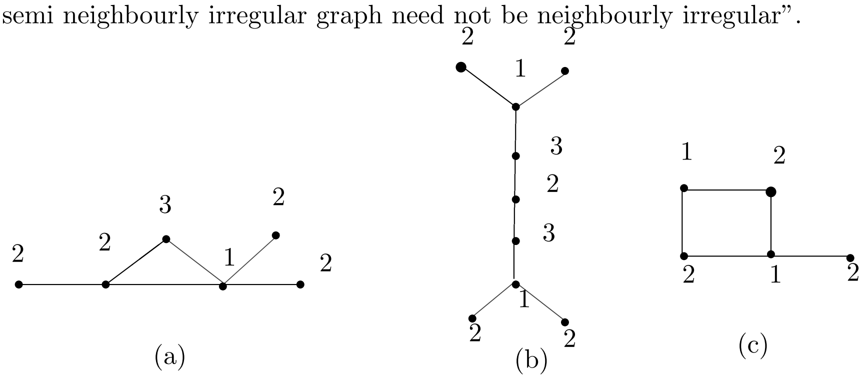

Figure 1.

Theorem 2.5. *For any $n \geq 3$, there exists at least one semi neighbourly irregular graph of order n.*

Proof. Every positive integer $n \geq 3$, has a partition $(1, n-1)$. Therefore, every positive integer $n \geq 3$, has at least one partition with distinct parts. For each partition with distinct parts, there exists a complete partite graph. Thus, for any $n \geq 3$, there exists at least one $K_{n_1,n_2,\ldots,n_m}$ such that all n_i's are distinct. □

Remark 2.6. The class of all $K_{n_1,n_2,\ldots,n_m}$ graphs is only a proper subclass of the class of all semi neighbourly irregular graphs. For example Figure 2 is semi neighbourly irregular of order 5, which is not in the class of all $K_{n_1,n_2,\ldots,n_m}$ graphs.

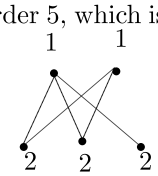

Figure 2.

Next we will see some other class of semi neighbourly irregular graphs which are not in the class of all $K_{n_1,n_2,\ldots,n_m}$ graphs.

3. $S_{m,t}$ - Graphs

Definition 3.1. Let $S_{m,t}$ denote the bipartite graph of order n having distinct partite sets $V_1 = \{u_1, u_2, \ldots, u_m\}$ and $V_2 = \{v_1, v_2, \ldots, v_t\}$, where $m < t$, and edge set $E(S_{m,t}) = \cup_i E_i$, where $E_i = \{u_i v_j / m - i + 1 \leq j \leq t, \text{and} 1 \leq i \leq m\}$.

By construction of $S_{m,t}$, $d_2(u_i) = m-1$, $(i = 1, 2, \ldots, m)$ and $d_2(v_i) = t-1 (i = 1, 2, \ldots, t)$ with $m < t$. Therefore $S_{m,t}$ is SNI graph of order n, where $n = m + t$ and $m < t$.

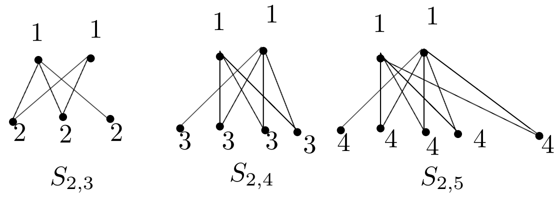

Figure 3.

Remark 3.2. For each odd n, $((n-1)/2)S_{m,t}$ graphs exist. For each even n, $((n-2)/2)$ $S_{m,t}$ graphs exist.

Theorem 3.3. *For any $n \geq 3$, there exist at least one $S_{m,t}(m < t)$ of order $n(= m+t)$.*

Proof. Since every positive integer $n \geq 3$, has at least one partition with distinct parts. □

Definition 3.4. Gear graph is obtained from the wheel W_n $(n \neq 4)$ by inserting a vertex between every pair of adjacent vertices of the cycle.

Proposition 3.5. *Gear graph is semi neighbourly irregular for $n \neq 4$.*

Proof. Let $V(W_n) = \{v_1, v_2, v_3, \ldots, v_n\} \cup \{v\}$. $E(W_n) = E(C_n) \cup \{vv_i/(1 \leq i \leq n)\}$. Inserting u_i between v_i and v_{i+1} and u_n between v_n and v_1, vertex set of Gear graph is $\{v_1, v_2, v_3, \ldots, v_n\} \cup \{v\} \cup \{u_1, u_2, u_3, \ldots, u_n\}$ and edge set is $\{vv_i/(1 \leq i \leq n)\} \cup \{v_iu_i/(1 \leq i \leq n)\} \cup \{u_iv_{i+1}/(1 \leq i \leq n-1)\} \cup \{u_nv_1\}$. Here $d_2(v_i) = n-1$, for $(1 \leq i \leq n)$, $d_2(u_i) = 3$, for $(1 \leq i \leq n)$, and $d_2(v) = n$. No two adjacent vertices have same d_2. Therefore Gear graph is semi neighbourly irregular for $n \neq 4$. □

Remark 3.6. Following graphs are not semi neighbourly irregular:

(1) $K_{n_1,n_2,\ldots,n_m}$ with atleast two n_i's the same.
(2) Any path P_n $(n \neq 3)$.
(3) Any cycle C_n $(n \geq 3)$.
(4) All complete graphs.
(5) Helm graph
(6) Web graph

Observation 3.7

i. If any vertex v in a graph G is adjacent with vertex u of degree n and non adjacent with vertices which are adjacent with u, then v is at a distance two away from at least $n-1$ vertices.

ii. If a graph G is semi neighbourly irregular, and it contains paths P_4, then no P_4 (path on 4 vertices) contains internal vertices having same number of vertices at a distance two away from them.

4. Semi neighbourly irregular graphs containing a graph as an induced subgraph

In this section, we construct semi neighbourly irregular graphs containing the given graph as an induced subgraph and also we define semi neighbourly regular strength of G.

Theorem 4.1. *There exists a semi neighbourly irregular graph of order* $4n-1$*, containing every graph of order* $n \geq 2$ *as an induced subgraph.*

Proof. Let G be any graph of order n. Let $V(G) = \{v_i/(1 \leq i \leq n)\}$. The desired graph S has the vertex set $V(S) = V(G) \cup V(T) \cup V(W)$, where $V(T) = \{t_i/(1 \leq i \leq n)\}$ and $V(W) = \{w_i/(1 \leq i \leq 2n-1)\}$. Let $E(S) = E(G) \cup \{v_i t_i/(1 \leq i \leq n)\} \cup \{t_i w_j/(1 \leq i \leq n), (1 \leq j \leq n-1+i)\} \cup \{v_j t_i/(1 \leq i \leq n, i+1 \leq j \leq n$ and $v_j v_i \notin E(G)\}$. Here, we observe that $d_2(v_i)$ in $S = 2n-2+i$, for $(1 \leq i \leq n)$, $d_2(w_i)$ in $S = 3n-2$, for $(1 \leq i \leq n)$, $d_2(w_{i+n-1})$ in $S = 3n-i-1$, for $(2 \leq i \leq n)$; $d_2(w_i)$ in S not equal to $d_2(t_i)$ in S; for $(1 \leq i \leq n)$ and $d_2(w_{i+n-1})$ in S not equal to $d_2(t_i)$ in S, for $i = 2, 3, 4, \ldots, n$. $d_2(v_i)$ in S not equal to $d_2(t_i)$ in S, for $i = 2, 3, 4, \ldots, n$. Therefore, the desired graph S is the semi neighbourly irregular graph of order $4n-1$ containing every graph of order $n \geq 2$ as an induced subgraph. The number of edges in the graph S is $2n^2$. □

5. Minimal Edge Covering

A family of edges of a graph is called a covering edge family when it has at least one edge at each vertex.

Theorem 5.1. *A minimal covering edge family of semi neighbourly irregular graph* S *of order* $4n-1$ *containing given graph of order* $n \geq 1$ *as an induced subgraph has cardinality* $3n-1$.

Proof. Let$w_1, w_2, w_3, \ldots, w_n, w_{n+1}, \ldots, w_{2n-2}, w_{2n-1}; t_1, t_2, t_3, \ldots, t_n$ and $v_1, v_2, v_3, v_4, \ldots, v_n$ be the vertices of semi neighbourly irregular graph S of order $4n-1$ containing any given graph of order $n \geq 1$ as an induced subgraph.
Let E_1 be the set of edges $t_1w_1, t_1w_2, t_1w_3, \ldots, t_1w_{n-1}, t_1w_n, t_2w_{n+1}, t_3w_{n+2}, \ldots, t_nw_{2n-1}$.

This set E_1 covers all the vertices $w_1, w_2, w_3, w_4, \ldots, w_n, w_{n+1}, \ldots, w_{2n-2}, w_{2n-1}$ and $t_1, t_2, t_3, \ldots, t_n$. The remaining vertices $v_1, v_2, v_3, \ldots, v_n$ are covered by the edges $t_1v_1, t_2v_2, t_3v_3, \ldots, t_nv_n$. Therefore a minimal covering edge family of semi neighbourly irregular graph S of order $4n-1$ containing given graph of order n as an induced subgraph has cardinality $3n-1$. □

6. Minimal Vertex Covering

Minimum number of vertices which covers all the edges of a graph is called minimal vertex covering number.

Theorem 6.1. *A minimal vertex covering number of semi neighbourly irregular graph* S *of order* $4n-1$ *containing given graph of order* n *as an induced subgraph is* $2n-1$.

Proof. The vertices $t_1, t_2, t_3, \ldots, t_n$ cover all the edges in the sets $\{v_it_i/(1 \le i \le n)\} \cup \{t_iw_j/(1 \le i \le n), (1 \le j \le n-1+i)\}$ and remaining edges are covered by $n-1$ vertices of $v_1, v_2, v_3, v_4, \ldots, v_n$. Hence a minimal vertex covering number of semi neighbourly irregular graph S of order $4n-1$ containing given graph of order $n \ge 1$ as an induced subgraph is $2n-1$. □

Remark 6.2. We observe that semi neighbourly irregular graph containing K_2 as an induced subgraph is P_3 and is of order 3 and semi neighbourly irregular graph containing $K_{2,2}$ as an induced subgraph is of order 5. But in the above construction, semi neighbourly irregular graph containing K_2 as an induced subgraph is of order 7 and semi neighbourly irregular graph containing $K_{2,2}$ as an induced subgraph is of order 15.

Now we will determine minimum number of points needed to construct semi neighbourly irregular graph containing some particular graphs.

Theorem 6.3. *For $n \ge 1$, the minimum order of semi neighbourly irregular graph containing the regular complete bipartite graph $K_{n,n}$ of order $2n$ as an induced subgraph is $2n+1$.*

Proof. By attaching one pendant vertex to any one of the vertex of $K_{n,n}$ we get semi neighbourly irregular graph of order $2n+1$. This attachment makes d_2 of each vertex in one partition is n. d_2 of each vertex in another partition set is $n-1$. □

Theorem 6.4. *For any $n > 3$, every n-cycle is an induced sub graph of semi neighbourly irregular graph of order $2n$.*

Proof. Case (i) : n is even ($n \ge 4$).
By attaching two pendant vertices to alternate vertices of n -cycle (n is even) we get SNI graph of order $2n$.

Case (ii) : n is odd ($n \ge 5$).
Let C_n be an odd n-cycle. $V(C_n) = \{v_1, v_2, \ldots, v_n\}$. By attaching three pendant vertices to v_1, and attaching two pendant vertices to v_i, for $i = 3, 5, 7, \ldots, n-2$, we get semi neighbourly irregular graph of order $2n$. □

Theorem 6.5. *For any $n \ge 3$, every path P_n is an induced sub graph of semi neighbourly irregular graph (tree) of order $2n-3$.*

Proof. Case (i): n is odd.
If $n = 3$, P_3 is semi neighbourly irregular graph. Let P_n ($n \ge 5$) be a path of length $n-1$. $V(P_n) = \{v_1, v_2, v_3, \ldots, v_n\}$. By attaching one pendant vertex to v_2 and attaching one pendant vertex to $(n-1)^{st}$ vertex of P_n and then attaching two pendant vertices to v_i, for $i = 4, 6, 8, \ldots, n-3$, we get semi neighbourly irregular graph (tree) of order $2n-3$.

Case (ii): n is even.
If $n = 4$, attach one pendant vertex to v_2, to get semi neighbourly irregular tree of smallest order 5. Let $P_n (n \ge 6)$ be a path of length $n-1$. $V(P_n) = \{v_1, v_2, v_3, \ldots, v_n\}$.

By attaching one pendant vertex to v_2, and attaching two pendant vertices to v_i, for $i = 4, 6, 8, \ldots, n-2$, we get semi neighbourly irregular graph(tree) of order $2n-3$. □

Theorem 6.6. *For $n \geq 2$, the smallest order of semi neighbourly irregular graph containing K_n as an induced subgraph is of order $2n-1$.*

Proof. Let K_n be the complete graph of order $n \geq 2$. If $n = 2$, then the graph P_3 is semi neighbourly irregular graph containing K_2. We assume that $n \geq 3$. Let $V(K_n) = \{v_1, v_2, \ldots, v_n\}$. To the graph K_n, we add $(n-1)$ new vertices $u_1, u_2, u_3, \ldots, u_{n-1}$ to $V(K_n)$ and we add several additional edges to complete the construction of G.

For $1 \leq i \leq n$, we join v_i and u_j with $1 \leq j < i$. That is, $E(G) = E(K_n) \cup \{v_iu_j/1 \leq i \leq n, 1 \leq j < i\}$. Moreover, $d_2(v_i) = n - i$, for $(1 \leq i \leq n)$ and $d_2(u_i) = n + i - 2$, for $(1 \leq i \leq n-1)$. Therefore, no two adjacent vertices have same number of vertices at a distance two away from them. The resulting graph G is a semi neighbourly irregular graph which contains K_n as an induced subgraph. We denote this graph by SNI K_n.

Also, we observe that $d(v_i)$ in G is $n+i-2$, $(1 \leq i \leq n)$, $d(u_i)$ in G is $n-i$, $(1 \leq i \leq n-1)$. The degree sequence is $(1, 2, 3, \ldots, [(2n-1)/2], [(2n-1)/2], \ldots (2n-2))$. The d_2 sequence is $0, 1, 2, 3, \ldots, [(2n-1)/2], [(2n-1)/2], \ldots, (2n-3)$. This graph has exactly two vertices of same d_2. If we attach one pendant vertex to the vertex which has $d_2 = 0$ (or maximum degree vertex) then we have semi neighbourly irregular graph of even order. By attaching n-pendant vertices to the vertex which has zero d_2 (maximum degree), then we get semi neighbourly irregular graph and d_2 of each vertex is increased by n. □

For every graph G, minimum number of points needed to construct semi neighbourly irregular graph containing G to be considered and we will define semi neighbourly regular strength of G.

Definition 6.7. Let G be a graph with n vertices. The semi neighbourly regular strength $(SNRS)$of G is the minimum number k such that G is an induced subgraph of semi neighbourly irregular graph.

Proposition 6.8. *$SNRS(C_n) = n$, $SNRS(P_n) = n-3$, $SNRS(S) = 0$, $SNRS(K_{n,n}) = 1$, $SNRS$ of SNI graphs is 0 and $SNRS(K_n) = n-1$.*

7. Semi Neighbourly Irregular Trees

In this section, we define semi neighbourly irregular tree and strict semi neighbourly irregular tree and include a few properties possessed by semi neighbourly irregular trees.

Now we recall the definitions, (i) The graph $S(G)$ obtained from a graph G by subdividing each edge of G exactly once is called the Subdivision of G. (ii) K_2 with n pendant edges at each end point is called the Bistar (or) Barbell graph and is denoted by $B_{n,n}$. (iii) K_2 with n pendant edges at one end point and m pendant edges at another end point is called $B_{n,m} (n \neq m)$ Star (tree).

Definition 7.1. A tree T is said to be semi neighbourly irregular (SNI) tree if no two adjacent vertices of T have the same number of vertices at a distance two away from them.

Proposition 7.2. *For any $n \geq 3$, there exists a semi neighbourly irregular Star, of order n.*

Proof. Any $n \geq 3$ can be partitioned into 1 and $n-1$. By Theorem 2.3, for any $n \geq 3$, $K_{1,n-1}$ is SNI star. Size of the star is $n-1$. □

Proposition 7.3. *Subdivision of $K_{1,n}(n \geq 3)$ is semi neighbourly irregular tree.*

Proof. Let $K_{1,n}(n \geq 3)$ be a star. Let u be the center vertex of degree n. Let $v_1, v_2, \ldots, v_n$ are vertices adjacent with u. Subdividing each edge of $K_{1,n}$ one time, we get $S(K_{1,n})$. We get n vertices $w_1, w_2, w_3, \ldots, w_n$ such that each w_i is adjacent with center vertex u and adjacent with v_i. $d_2(u) = n, d_2(v_i) = 1$, $i = 1, 2, \ldots, n$ and $d_2(w_i) = n-1$, $i = 1, 2, \ldots, n$. Therefore $S(K_{1,n})$ is semi neighbourly irregular tree. Size of this graph is $2n$. □

Proposition 7.4. *Subdivision of $B_{n,n}, (n \geq 2)$ is semi neighbourly irregular tree of order $4n + 3$.*

Proof. Let $V(K_2) = \{v_1, v_2\}$. Let u_i $(1 \leq i \leq n)$ be the vertices adjacent to v_1 and w_i $(1 \leq i \leq n)$, be the vertices adjacent to v_2. Subdivide each edge of $B_{n,n}$ one time, we get $S(B_{n,n})$. In $S(B_{n,n})$, for each edge $v_1u_i(1 \leq i \leq n)$, we get n vertices $x_1, x_2, \ldots, x_n$ such that each x_i is adjacent with v_1 and u_i and for each edge $v_2w_i(1 \leq i \leq n)$, we get n vertices $y_1, y_2, \ldots, y_n$ such that each y_i is adjacent with v_2 and w_i. For the edge v_1v_2, we get a new vertix x which is adjacent to both v_1 and v_2. For $i = 1, 2, \ldots, n$, $d_2(u_i) = 1$ and $d_2(x_i) = n$, $d_2(v_1) = n + 1$, $d_2(w_i) = 1$ and $d_2(y_i) = n$, $d_2(v_2) = n+1$, $d_2(x) = 2n$. Therefore, $S(B_{n,n})$ is semi neighbourly irregular tree of order $4n + 3$. □

Proposition 7.5. *Subdivision of $S(K_{1,n})$ is not semi neighbourly irregular.*

Proposition 7.6. *Subdivide K_2 in Bistar $B_{n,n}(n \geq 2)$. We get semi neighbourly irregular tree of order $2n + 3$.*

Proof. Let $V(K_2) = \{v_1, v_2\}$. Let $u_i(1 \leq i \leq n)$ be the vertices adjacent to v_1 and $w_i(1 \leq i \leq n)$ be the vertices adjacent to v_2. Subdivide v_1v_2, we get a new vertex x which is adjacent to both v_1 and v_2. Then $d_2(v_1) = 1$, $d_2(v_2) = 1$, $d_2(u_i) = n$, for $(1 \leq i \leq n)$ and $d_2(w_i) = n$, for $(1 \leq i \leq n)$, $d_2(x) = 2n$. Therefore, we get semi neighbourly irregular tree of order $2n + 3$. □

Proposition 7.7. *$B_{n,m}$ $(n \neq m)$ Star (tree) is semi neighbourly irregular.*

Proof. Let $V(K_2) = \{v_1, v_2\}$. Let $v_i, (1 \leq i \leq n)$ be the vertices adjacent with v_1 and non adjacent with v_2 and $u_i(1 \leq i \leq m)$ be the vertices adjacent with v_2 and non adjacent with v_1. Therefore $d_2(v_1) = m$, $d_2(v_i) = n$, for $1 \leq i \leq n$, $d_2(v_2) = n$, $d_2(u_i) = m$, for $1 \leq i \leq m$. Hence $B_{n,m}$ is SNI tree of order $2 + m + n$. □

Definition 7.8. A semi neighbourly irregular tree T is called strict semi neighbourly irregular tree if removal of any pendant vertex in T results in a non semi neighbourly irregular tree.

Example 7.9. The following graphs are strict semi neighbourly irregular trees.

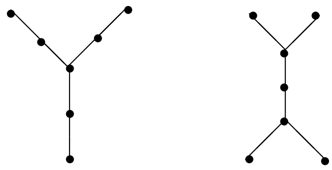

Figure 5.

Remark 7.10. Cartesian product of semi neighbourly irregular graphs need not be a semi neighbourly irregular graph. From figure 5, we can see that P_3 is SNI but $P_3 \times P_3$ is not SNI.

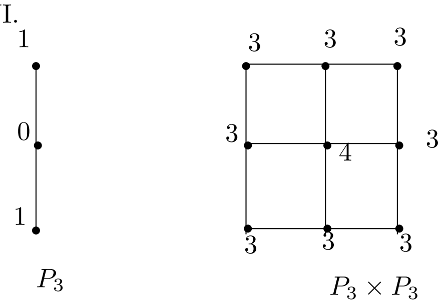

Figure 4.

References

1. Northup,A.: A Study of semiregular Graphs, Preprint, 2002.
2. Bloom,G.S., Kennedy,J.K., Quintas,L.V.: Distance degree regular graphs, *The Theory and Applications of Graphs,* Wiley, New York, (1981) 95-108.
3. Bondy,J.A. , Murty,U.S.R.: *Graph Theory with Application*, MacMillan, London ,1979.
4. Harary, F.:*Graph Theory*, Addison - Wesley, 1969.
5. Prakasam,S.G., Ayyaswamy,S.K.: Neighbourly Irregular Graphs,*Indian J. pure appl. Math.*, **35**(3)(2004)389-399.
6. Erdos,P., P.J. Kelly,P.J.: The minimal regular graph containing a given graph, *Amer. Math. Monthly*, **70**(1963)1074-1075.
7. Konig,D.: Theoritic der Endlichen and Unendlichen Graphen, Leipizig ,1936.

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